intended to convey to the mind a clearer notion of the space relations than do the Mongean pictures. The authors have not attempted to explain the methods of constructing such figures and there can be no objection to this in a book which concerns itself primarily with the Mongean method. However, such pictures, when used, should be properly drawn. In Fig. 96, for instance, the ellipses representing the circles should not have their principal axes horizontal and vertical, and the lines cc^v and bb^v should be tangent to the upper ellipse and would be if that ellipse were properly drawn. The same remarks apply to the lower ellipse and to certain other figures.

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Plane and Spherical Trigonometry, with Tables. By G. N. BAUER and W. E. BROOKE. Second Revised Edition. D. C. Heath and Co., Boston, 1917. xi + 174 + v + 139 pages.

A writer of textbooks may approach trigonometry from two different standpoints; he may wish to give merely the formulas and processes needed in the solution of triangles, or he may regard trigonometry as a chapter of mathematics dealing with a certain well-defined class of functions whose use in solving triangles is incidental. If in American colleges we could separate our engineering students from the specialists in mathematics, two different types of textbook could be devised to meet these two tendencies; this being as a rule impossible, an attempt must be made to meet both requirements at once. A glance at recent texts shows that we have apparently arrived at a sort of standard in the choice of subject matter; to summarize briefly, an acceptable textbook on trigonometry should contain, it would seem, the following topics:

(*a*) The definitions and elementary relations of the six functions;
(*b*) The addition-formulas and factor-formulas for changing sums of sines or cosines into products;
(*c*) The formulas for $2x$ and $x/2$, or even $3x$, but with no reference to wider formulas of which these might be the simplest cases;
(*d*) The graphs of the functions, either in a separate chapter or throughout;
(*e*) The treatment of trigonometric equations, at least among the exercises, and preferably with ample use of graphic methods;
(*f*) The application of formulas to the solution of plane and spherical triangles.

Besides these absolute essentials, we usually find chapters on

(*g*) The inverse functions;
(*h*) De Moivre's Theorem, and its simpler consequences; such as for instance the formulas for sin (nx), and the definitions of the hyperbolic functions, with their graphs perhaps.

Since trigonometry is usually a 3-hour branch for one semester, these matters can not be handled satisfactorily to any extent; a choice must be made, and here is where the difference in textbooks comes in.

Our present text, under (*a*), defines the 6 functions once and for all by means of the coördinates of the endpoint P of a rotating line-segment pivoted at the origin; the first 50 pages develop this topic splendidly, after which 5 more pages are given to the line-values of the functions in connection with a unit circle. The exercises are well selected; nothing but praise can be bestowed upon this treatment (Chap. 1–5).

Concerning (*b*), we find the addition-formulas proved by the trick device

$$\sin (x + y) = \frac{BQ + RP}{OP} = \frac{BQ}{OQ}\frac{OQ}{OP} + \frac{RP}{QP}\frac{QP}{OP},$$

and we are moved to enter a mild protest; such a proof is of no value to the student. There are many ways of proving this relation without having recourse to such artificial transformations. In extending the formula to angles beyond 90°, the authors have followed the usual plan of first putting

$$\sin (x + y + 90°) = \cos (x + y),$$

and then, in the expansion, replacing $\cos x$ by $\sin (90° + x)$, etc. The exercises include the formulas for the functions of $3x$. The reduction of $\sin (mx) \cos (nx)$ and similar forms to a sum of sines or cosines is mentioned briefly, but clearly. The cases of trigonometric equations which are presented for solution are to be found here and there scattered among the other exercises; they are sufficient in number and variety, perhaps; but would it not have been better to concentrate this entire matter in one chapter, with more explanation, and with emphasis on graphical methods?

Speaking of graphs, our authors have conceded to them the utmost minimum of space and importance; out of 125 pages devoted to plane trigonometry, only two pages are assigned to the graphs of the functions, with a grand total of six examples to be used as material for exercises! Surely this procedure is open to grave criticism and is counter to the spirit of the times. Skillful handling of equations necessarily demands graphic methods; and when we reflect that popular magazines nowadays are furnishing plans and diagrams for home-made harmonic-motion machines, when we think of alternating currents with their trigonometric equations, or when we consider the solution of mixed equations like $x^2 - 1 + 2 \sin x = 0$, we can only wonder why the authors have practically excluded graphic methods from their text. Where shall we find a treatment of trigonometric graphs if not in a trigonometry? This omission gives the book an old-fashioned appearance, and might prejudice a casual reader at the first glance. In this connection we may note that the unshaded, light-line diagrams used on pp. 128, 135 in illustration of spherical triangles are unsatisfactory to the eye through want of perspective.

The treatment of spherical triangles is adequate for practical purposes; the only area-formula mentioned is the one employing the angles as such, and no allusion is made to the existence of sine or tangent formulas for the area. Here and elsewhere the authors have not realized how the dry and didactic tone can

be relieved and how the student's horizon can be widened by occasional references to other methods and formulas, as well as by slight historical excursions now and then. The presentation of the triangle-solutions, both plane and spherical, is carried out well; the pupil will learn how to conduct his work neatly and economically by studying the typical solutions of the text.

So much for the essential contents of this little volume. A rather advanced chapter on inverse notation and functions is inserted immediately after the addition-formulas; the average student will not master this, but it is there if needed. The chapter on De Moivre's theorem concerns itself chiefly with the nth roots of unity and of complex numbers generally, and with the development of the sine and cosine series; these are found by first getting sin (nx) from De Moivre's theorem, putting $nx = y$, and then keeping y constant while x approaches zero. There is ample authority for this method; but will it make an appeal to the beginner? The matter of roots belongs rather to algebra; a more trigonometric use of the theorem might no doubt have been made by developing the formulas given in Chrystal's *Algebra*, Vol. 2, pp. 275–278, which are moreover of great importance in trigonometric integration.

The tables at the end give the natural functions to 3 and then to 4 places; and their logarithms, as also those of the natural numbers, to 3, 4, and 5 places in succession. All these are very clear and legible, with the exception of Table IX; here the columns of difference-numbers are placed in between the vertical columns for sine, cosine, and tangent, and moreover they are placed halfway between the logarithmic numbers. Since there are no horizontal or vertical guidelines to lead the eye, a zigzag effect is produced which is very annoying to one's eyesight and patience. This ought to be corrected when the time comes for another edition.

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BOOK NOTICES.

It is now rather usual for a textbook on "Analytic Geometry" to contain a chapter on "Empirical Equations." The new *Analytic Geometry* just published by Edwin S. Crawley and Henry B. Evans of the University of Pennsylvania is no exception. It contains such a chapter of fourteen pages. This is about the usual length.

A more extensive treatment of the subject of empirical equations is to be found in the book entitled *Empirical Formulas* by Theodore R. Running, associate professor of mathematics in the University of Michigan, recently published by John Wiley and Sons as No. 19 of the series of Mathematical Monographs edited by Mansfield Merriman and Robert S. Woodward.

Doctors' theses which are not printed in mathematical periodicals are likely to be overlooked by some who would be interested in them. Two such theses submitted to the faculty of the Catholic University of America have recently